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ABSTRACT

Designed for the student who has mastered the skills in Math Structures 1 and 2 (of the Quinmester Program), this course includes work with flow charts, sequences and series, Pascal's triangle, magic squares, number patterns, similar figures, and coding. Each topic has a suggested time limit, performance objectives, teaching suggestions, and textbook references. Overall goals are given for the course and a sample posttest is included, along with an annotated bibliography of 30 references. (DT)

**AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM**

**PATTERNS IN MATHEMATICS**

5212.65
5214.65



DADE COUNTY PUBLIC SCHOOLS

DIVISION OF INSTRUCTION • 1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

# PATTERNS IN MATHEMATICS

5212.65
5214.65

(EXPERIMENTAL)

Written by

William F. Younkin

for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
1971-72

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## PREFACE

In an effort to provide for different interests in the various schools, the following course of study has been designed to include more topics than a class can be expected to cover in one nine-week quinmester. A time allotment for each topic is suggested as an aid in planning. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

## CATALOGUE DESCRIPTION

A non-rigorous investigation of the mathematics of patterns. Reinforces, fundamental skills in computing with rational numbers. Includes sequences, series, Pascal's triangle, square numbers, magic squares, codes, decoding, and use of flow charts.

Designed for the student who has mastered the skills described in Math Structures 1 and 2.

## TABLE OF CONTENTS

## OVERALL GOALS

The student will

1. Grow in his ability to recognize patterns, isomorphisms and symmetries.

2. Gain experience in informally formulating and validating generalizations from data given.

3. Demonstrate his interest by participating in class discussions, group activities, and completing at least one independent project utilizing patterns.

4. Improve and extend his mastery of the basic operations with rational numbers.

KEY TO STATE ADOPTED TEXTS

SM(I)
SM(II) - Eicholz, Robert; O'Daffer, Phares; Brunfiel, Charles; Shanks, Merrill; Fleenor, Charles R. School Mathematics I and II. Palo Alto, California: Addison-Wesley Publishing Co., 1967.

Foley, Jack; Jacobs, Wayne; and Basten, Elizabeth. Individualizing Mathematics. Menlo Park, California: Addison-Wesley Publishing Company, 1970.

F-SP-FC(I) - Skills and Patterns: Flow Chart I
F-SP-N - Skills and Patterns: Number, Patterns -Theory
F-PD-FC(II) - Patterns and Discovery: Flow Chart II
F-DS-FC(III) - Discovery and Structure: Flow Chart III
F-DS-NT - Discovery and Structure: Number Theory, Patterns and Number Forms
F-DS-S - Discovery and Structure: Similarity

AIM(1,P) - Johnson, D. A.; Hansen, V. P.; Peterson, W. H.; Rudnick, J. A.; Cleveland, R.; and Bolster, L. C. Activities in Mathematics: First Course - Patterns. Glenview, Illinois: Scott, Foresman and Co., 1971.

AIM(2,Gr) - Johnson, D. A.; Hansen, V. P.; Peterson, W. H.; Rudnick, J. A. Cleveland, R.; and Bolster, L. C. Activities in Mathematics: Second Course - Graphs. Glenview, Illinois: Scott, Foresman and Co., 1971.

Mc(7) - McSwain, E. T.; Brown, K. E.; Gundlach, B. H.; and Cooke, R. J. Mathematics 7. River Forest, Illinois: Laidlaw Brothers, 1965.

SMS(2) - Suppes, Patrick; Meserve, Bruce; and Sears, Phyllis. Sets, Numbers, and Systems, Book 2. New York: The L. W. Singer Company, Inc., 1970.

Note: Other references are identified by the number of the item in the bibliography followed by the name of the author in parenthesis. For example,

22 (Krulik)

refers to item number 22 on the third page of the Annotated Bibliography at the end of this course of study.

OBJECTIVES, STRATEGIES AND SOURCES

I. FLOW CHARTS (1 week)

PERFORMANCE OBJECTIVES

The student will

1. Follow a flow chart of an unfamiliar activity, problem or skill to a desired conclusion.

2. Construct a simple flow chart for
   a. A non-mathematical procedure
   b. A simple mathematical problem

| Strategies | References (*state adopted) |
| --- | --- |
| A. Teacher-directed introduction | |
| 1. What is a flow chart? | |
| 2. Why use a flow chart? | 22 (Krulik) |
| a. Advantages | |
| b. Disadvantages | |
| 3. Why learn to make a flow chart? | |
| B. How to read a flow chart | *F-SP-FC(I) |
| 1. Exhibit several charts | |
| a. On posters | *AIM(1,P) |
| b. On overhead transparencies | pp. 15-22 |
| c. On the board | |
| 2. Discuss standard symbols | *AIM(2,Gr) |
| 3. Use a variety of worksheets | pp. 71-74 |
| a. May be based on F-SP-FC(I) | |
| b. May be based on original flow chart developed by class | |
| C. How to make a flow chart | *F-SP-FC(I) |
| 1. Provide a set of statements, not in proper order | |
| a. Students create flow chart | |
| (1) Together in groups | |
| (2) Individually | |
| b. Students exchange flow charts | |
| (1) Check by trying to follow steps | |
| (2) Make revisions where needed | |
| 2. Suggest a topic (for individual or group work) | |
| a. Student lists steps in statement form | |
| b. Student makes flow chart | |
| c. Let students exchange charts to check them | |

I. Flow charts (continued)

| Strategies | References (*state adopted) |
|---|---|
| D. Practical applications of flow charts | *F-PD-FC(II)<br>*F-DS-FC(III) |

E. Individual Problems
1. With suggestions from students make a list of topics
2. Student selects topic from suggested list
   a. Creates flow chart
   b. Tries it out on a fellow student
3. Student projects may include
   a. Poster-size flow chart
   b. Booklet with several original flow charts
4. Student projects may be long range
   a. On topics discussed in class later
      (1) Solving series
      (2) Pascal's triangle
   b. On topics not discussed in class

F. Added notes
1. Flow-charting can become boring if the work is not varied
2. If not available locally, templates may be purchased from

   Olivetti Underwood Corp.
   One Park Avenue
   New York, N.Y. 10016
3. Some non-mathematical topics to be charted are
   a. How to sharpen a pencil.
   b. How to open a door.
   c. How to lead a cheer.
   d. How to find a specific page in a book.

## OBJECTIVES, STRATEGIES AND SOURCES

II. SEQUENCES AND SERIES (3 weeks)

### PERFORMANCE OBJECTIVES

The student will

1. Devise a method of generating successive terms of an arithmetic sequence and compute the next four terms, when given the first four terms of the sequence.

2. Compute the four successive terms of a Fibonacci sequence and explain his computational method, when given the first five terms of the sequence.

3. Express any square number as a series of odd numbers.

4. Determine the Nth triangular number from a series of N natural numbers.

5. Devise, state, and use a rule for computing Y when X is known, given a table of related values of X and Y.

6. Devise a method of determining the successive terms of a sequence described by a second degree equation and compute the next three terms, when given the first four of the sequence.

7. Study a sequence of three geometric figures then:
    a. draw the next three figures in the sequence
    b. compute the perimeter and area of each new figure
    c. determine the perimeter and area of the 7th, 8th, and 9th figures in the sequence without drawing the figures.

### Strategies

A. Arithmetic sequences
   1. Introduction
      a. What is an arithmetic sequence?
      b. How can I determine the next number in the sequence?
      c. How can I determine the operations to use?
      d. Place a large number of sequences on the board and have students solve them individually.

II. Sequences and Series (continued)

A. Arithmetic sequences (continued)

| Strategies | References (*state adopted) |
|---|---|
| 2. Students need practice in determining the rule | |
| a. "Manufacturing numbers" helps to give the student a basis for finding a general rule. | *AIM(2,Gr) pp. 61-66 |
| b. What's My Rule? - an excellent game as a whole-class activity or for small groups | *AIM(2,Gr) pp. 67-74 |
| 3. Define an arithmetic series as the sum of the terms of an arithmetic sequence. Use finite series to develop formula for finding the sum $S_n = \frac{n}{2}(a_1 + a_n)$ where n = the number of terms, $a_1$ = the first term, $a_n$ = the nth term | 30(Wiebe) pp. 362-376 |
| 4. Make a distinction between a sequence and a series; for example $1, 5, 9, \ldots, 4n - 3, \ldots$ is a sequence, while $1 + 5 + 9 + \ldots + (4n - 3) + \ldots$ is a series. | |

B. Fibonacci numbers

| | |
|---|---|
| 1. After showing the first few terms, have students compute first 30 terms in the Fibonacci sequence and list in a table. | 11 (Bezuszka - 1) pp. 61-62 |
| 2. Investigate Fibonacci sequence | 30 (Wiebe) pp. 385-388 |
| a. Sum of consecutive terms | |
| b. Difference of consecutive terms | |
| c. Quotients of consecutive terms related to "Golden Mean" | 24 (Meyer) |
| 3. Investigate other Fibonacci patterns | 11 (Bezuszka - 3) p. 53 |
| a. Odd number triangle | |
| b. The Golden Section | |
| c. Pythagorean Triples | 27 (Tassone) |
| | 11 (Bezuszka - 2) pp. 49-51 |
| | 11 (Bezuszka - 3) pp. 53-57 |

II. Sequences and Series (continued)

| Strategies | References (*state adopted) |
|---|---|
| C. Figurate numbers | 11 (Bezuszka - 1) |
| 1. Square numbers | pp. 51-52 |
| a. Introduce geometrically | |
| b. Sum of consecutive odd numbers | 30 (Weibe) |
| c. Find the formula for square numbers | pp. 356-361 |
| 2. Triangular numbers | |
| a. Introduce geometrically | 16 (Edmunds) |
| b. Sum of consecutive natural numbers | pp. 104-105 |
| c. Compare sums of consecutive triangular numbers and consecutive square numbers | 11 (Bezuszka - 1) |
| d. Find formula for triangular numbers | pp. 49-51 |
| 3. Rectangular numbers | |
| 4. Other figurate numbers | 11 (Bezuszka - 1) |
| 5. Place square and triangular sequences one above the other | pp. 53-54 |
| a. Take successive differences of each sequence | 11 (Bezuszka - 2) pp. 56-57 |
| (1) What kind of sequences are they? | |
| (2) Compare the two | 11 (Bezuszka - 3) |
| (a) Term for term | p. 54 |
| (b) Make new sequences by finding difference of like terms. | 30 (Wiebe) |
| b. Make new sequence by finding differences between like terms of sequences. | pp. 373-376 |
| D. Equations | |
| 1. Review "What's My Rule" game | *AIM(2,Gr) |
| a. Formalize writing rule | pp. 61-74 |
| b. Compute y, given x and rule | |
| c. Graphing in 2-d | 16 (Edmonds) |
| Take care that <u>only natural numbers are used for x</u>. Recall that by definition, the domain of any sequence is the set of natural numbers. | pp. 53-67 |
| 2. Function machines | *SM(I) |
| a. Use of "machine" idea lays the foundation for increased understanding of the function concept which is vital to higher mathematics. | pp. 42-52, 252-258 |
| | *SM(II) |
| b. Student may design his own machine. | pp. 10-15 |
| c. Use machines to produce ordered pairs for a table and for graphing. <u>Stress that graphs will always be a set of unconnected points because of the restricted domain</u>. | 145-152, 419-421 |
| 3. Make flow charts for exercies on pages 67-68 in 16 (Edmonds). | |

II. Sequences and Series (continued)

<u>Strategies</u>

E. Finite Differences

1. Begin to formalize the methods developed earlier for finding successive terms of a sequence. Take the differences of successive terms of a sequence of five or more terms, thus forming another sequence.

   a. If the resulting sequence is constant, the value of any term ($t_n$) in the original sequence is given by the equation $t_n = an + b$, where $\underline{n}$ is the number of the term, $\underline{a}$ is the constant difference and $\underline{b}$ is the difference of the first term of the original sequence and the constant.

   Example:

   Sequence: 3, 7, 11, 15, 19,...

   Differences: 4, 4, 4, 4,...

$$t_n = an + b$$

$$\therefore\ t_n = 4n + (3-4)$$

$$= 4n - 1$$

$$t_3 = 4 \cdot 3 - 1 = 11$$

$$t_6 = 4 \cdot 6 - 1 = 23$$

   b. If the resulting sequence is not constant, take successive differences again to form a third sequence. If this third sequence is constant, the value of any term ($t_n$) in the original sequence is given by the equation $t_n = an^2 + bn + c$, where $\underline{n}$ is the number of the term, $\underline{a}$ is half the constant difference, $\underline{b}$ is the difference of the first term of the second sequence and three times half the constant difference, Strategies

and **c** is the difference of the first term of the original sequence and the sum of half the constant difference and **b**.

Example:

Sequence: 9, 21, 39, 63, 93,...

1st difference: 12, 18, 24, 30,...

2nd difference: 6, 6, 6,...

$$t_n = an^2 + bn + c$$

$$\therefore t_n = \frac{6}{2}n^2 + \left(12 - 3\cdot\frac{6}{2}\right)n + \left[9 - \left(\frac{6}{2} + 3\right)\right]$$

$$= 3n^2 + 3n + 3$$

$$t_2 = 3(2)^2 + 3\cdot 2 + 3 = 21$$

$$t_5 = 3(5)^2 + 3\cdot 5 + 3 = 93$$

2. This pattern can be continued if there are students with interest and good mathematical background.

| | References (*state adopted) |
|---|---|
| F. Sequences of geometric figures | |
| 1. Use SMS(2) as an introduction. | *SMS(2) |
| 2. On an individual or group basis, let students create new figure sequences. | pp. 378-379 |
| 3. Assign numbers to figures based on | 16 (Edmonds) |
| a. Length | 93-103 |
| b. Perimeter | |
| c. Area | |
| d. Numbers of sides | |
| 4. Discuss the kinds of sequences determined by 3a, b, c, and d. | |
| 5. For a given sequence, let student predict next number and then check by drawing figure. | |
| 6. Where possible, find defining term, $a_n$, so that a particular figure in the sequence may be drawn without drawing all preceding figures. | |
| 7. Relate sequence to limits. | *SMS(2) |
| a. Points on a number line | pp. 381-384 |
| b. Polygons inscribed in a circle | |
| (1) Mention Archimedes' Method of Exhaustion. | *SM(II) pp. 378-382 |
| (2) Approximate $\pi$ by drawing and measuring. | |

III. PASCAL'S TRIANGLE (1 week)

PERFORMANCE OBJECTIVES

The student will—

1. Build a Pascal triangle of 6 lines for a given generator.
2. Give the sum of the number in the nth line of a Pascal triangle generated by a, without building the triangle.
3. Exhibit one pattern that holds for any Pascal triangle.

| Strategies | References (*state adopted) |
|---|---|
| A. Introduce experimentally the | |
| 1. Possible combinations as the number of coins increases. | *29 (White) SMS(2) pp 421, 422 |
| 2. Number of ways N-members committees can be formed from a group of M people. | *SM(II) pp 347-349 |
| 3. Number of ways of placing N objects in M containers. | 15 (Edmonds) |
| 4. Binomial distribution machine (board with alternating rows of nails) | 9 (Adler) |
| B. Develop through patterns | 30 (Wiebe) pp 343-356 excellent - relates triangle to other patterns |
| 1. Diagonals | |
| a. Outside diagonals - 1 | |
| b. 2nd. diagonals - natural numbers | |
| c. 3rd diagonals - increase by successive natural numbers | |
| 2. Standard method | |

III. Pascal's Triangle (continued)

| <u>Strategies</u> | <u>References</u><br>(*state adopted) |
|---|---|
| C. Patterns | |
| 1. Sum of the nth line is $a \cdot 2^n$ where **a** is the generator | |
| 2. Sum of the diagonals gives Fibonacci numbers | |
| 3. Other patterns | 30 (Wiebe) pp.346-356 |
| D. Added notes | 9 (Adler) |
| 1. Good material for bulletin board displays | |
| 2. Good project material<br>a. report on Pascal<br>b. build a binomial machine | |
| 3. Film - Donald in Math Magic Land | |

## IV. MAGIC SQUARES (1 week)

### PERFORMANCE OBJECTIVES

The student will --

1. Build a 4 x 4 magic square when the magic constant is given.
2. Build a 3 x 3 magic square when the magic constant is given.

| Strategies | References (*state adopted) |
|---|---|
| A. Introduce through examples | |
| 1. Simple squares | 11 (Bezuszka) |
| 2. Magic triangles | |
| B. Methods of Construction | |
| 1. 3 x 3 squares | 21 (Keepes) |
| 2. Odd-celled squares | 8 (Adkins) |
| 3. Even-celled squares | |
| 4. Given the magic constant | 12 (Cappon) |
| C. Special Squares | |
| 1. Melancholia | 28 (Weiss), 24 (Meyer) |
| 2. Upside-down | 24 (Meyer) |
| D. Other Magic Figures | 11 (Bezuszka) |
| E. Added Notes | |
| 1. Good material for bulletin board displays | |
| 2. Good project material | |
| a. construction methods | |
| b. special squares | |
| 3. Don't over-use. Interested students will pick it up and run. Make resources available | |

## V. PATTERNS IN ARITHMETIC (2 weeks)

### PERFORMANCE OBJECTIVES

The student will --

1. Test for divisibility by 2, 3, 4, 5, 6, and 9 through use of divisibility rules.
2. Check multiplication problems.
3. Show the pattern that determines that $(-a)\cdot(-b) = ab$ when it is known that $(-a)\cdot b = -ab$.

| Strategies | References (*state adopted) |
| --- | --- |
| A. Computational Patterns | |
| 1. Number Palindromes | 11 (Bezuszka) |
| 2. Symmetry Patterns | " " |
| 3. Tautonyms | " " |
| 4. Lightning Calculator | 24 (Meyer) |
| 5. Multiples | |
| B. Divisibility | |
| 1. Introduce Divisibility Rules - what can we divide this number by evenly? | |
| 2. Factors | |
| 3. Odd/Even | *3 (Foley) |
| 4. Tests for 2, 3, 4, 5, 6, 9 | *6 (McSwain) |
| a. Patterns in addition and Subtraction | |
| b. Multiplication - Division Patterns | *4 (Johnson) |
| 5. Tests for other Primes | 25 (Smith) |
| 6. Tests in other Bases - relate to that base good extension to individual project | |

V. Patterns in Arithmetic (continued)

| Strategies | References (*state adopted) |
|---|---|
| C. Prime Numbers | |
| 1. Apply all Divisibility Rules | 11 (Bezuszka) |
| 2. Sieve of Erastosthenes | |
| a. Have students complete on prepared tables of numbers. | |
| 3. Goldbach's Conjecture | 30 (Wiebe) pp 382-385 |
| D. Casting Out Nines | |
| 1. Have students investigate excess of nines in numbers contained in a sum. | |
| 2. What is excess of nines in a multiple of nine? Divisibility rule relationship. | |
| 3. Develop checking by casting out nines. | 16 (Edmonds) |

## VI. SIMILAR FIGURES (1 week)

### PERFORMANCE OBJECTIVES

The student will —

1. Determine the ratio of the lengths of corresponding sides of two similar triangles whose dimensions are known.
2. Calculate the lengths of two sides of a triangle when the length of the third side and the lengths of all sides of a similar triangle are known.
3. Identify similar figures that are congruent by finding the ratio of the lengths of corresponding sides is 1:1.
4. Determine the actual lengths represented by any part of a scale drawing when given the key.

| Strategies | References (*state adopted) |
|---|---|
| A. Introduce equal ratios | |
| 1. Cross-product method | |
| 2. Give students two similar triangles and have them list all possible correspondences. Identify correspondence which results in equal ratios. | *3 (Foley - Similarity)<br>*7 (Suppes) |
| 3. Develop a formal definition of similar triangles. | |
| B. Define Congruence | |
| 1. Similar - ratio 1:1 | |
| C. Scales | |
| 1. Introduce map reading and scales (get Florida maps from service stations) | *3 (Foley - Measurement)<br>*7 (Suppes) |
| 2. Relate maps to similar figures. Scales = ratios | |

VI. Similar figures (continued)

C. Scales (continued)

| Strategies | References (*state adopted) |
|---|---|
| 3. Review measurement with rulers. | |
| 4. Review Approximation. | |

D. Indirect Measure

| | |
|---|---|
| 1. State seemingly impossible measurement problem (height of tall building, distance across stream) have students speculate as to possible solutions. | *3 (Foley - Measurement)<br>*7 (Suppes) |
| 2. Develop indirect measurement through similar triangles. | 30 (Wiebe)<br>pp 391-403 |

VII. CODING (3 days)

## PERFORMANCE OBJECTIVES

The student will —

1. Encode a simple message with a given coding pattern.
2. Decode a simple message with a given coding pattern.

| Strategies | References (*state adopted) |
| --- | --- |
| A. Introduce "Secret Codes" | |
| 1. Correspondence - set of letters - number sequence | |
| a. vary sequences | |
| 1. natural numbers<br>2. Fibonacci numbers<br>3. set of multiples | |
| 2. Have students make up their own codes. | |
| 3. Code - Wheel | *4 (Johnson) |
| 4. This has potential for student research project and report to class. | |

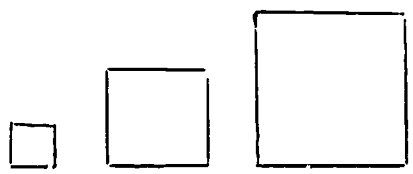
## SAMPLE POSTTEST ITEMS

List the next four terms for each sequence. Describe, in your own words, the pattern used to develop the sequence.

1. 0, 5, 10, 15, __, __, __, __.
2. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, __, __, __, __.
3. 0, 1, 4, 9, 16, __, __, __, __.
4. 1, 1, 2, 3, 5, __, __, __, __.

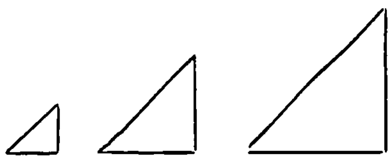
Draw the next three figures for each sequence.

5. 1 unit 1 square unit



1 2 3 4 5

6.



1 2 3 4 5

7. For the sequence of squares (in 5 above) fill in the chart

| square number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| perimeter | | | | | | | | |
| area | | | | | | | | |

PROJECTED

8. For the sequence of triangles, fill in the chart

| triangle number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| area | | | | | | | | |

For each chart, fill in the blanks and state a rule for computing Y given X.

9.

| X | Y |
|---|---|
| 1 | 3 |
| 2 | 4 |
| 3 | 5 |
| 7 | |
| 9 | |

10.

| X | Y |
|---|---|
| 1 | 5 |
| 2 | 7 |
| 3 | 9 |
| 4 | |
| 5 | |
| 10 | |

11.

| X | Y |
|---|---|
| 0 | 0 |
| 1 | 4 |
| 2 | 8 |
| 3 | 12 |
| 10 | |

12. Give the next four rows of Pascal's Triangle.

1 1

1 2 1

1 3 3 1

In your own words, state its significance.

13. Fill in the Magic Square.

| | | | |
|---|---|---|---|
| 10 | 3 | 15 | 6 |
| 5 | | | |
| | 13 | | 12 |
| 11 | 2 | | 7 |

14. Construct a Magic Square.

15. Complete the pattern:

$-2 \times 2 = -4$

$-2 \times 1 = -2$

$-2 \times 0 = 0$

$-2 \times -1 =$ __

$-2 \times -2 =$ __

$-2 \times -3 =$ __

In your own words, tell what the pattern tells you about $-a \times -b$.

16. Express the following square numbers as a sum of a series:

25 =

36 =

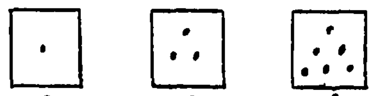
64 =

17. Determine the:

5th triangular number = _____.

7th triangular number = _____.

1st. 2nd. 3rd. triangular number.



1 3 6

18. State the divisibility rules for:

2

3

4

5

6

9

19. For each number below, state which members of the set $\{2, 3, 4, 5, 6, 9\}$ it is divisible by:

21 3144 186 351 450 1425

POSTTEST ANSWER KEY

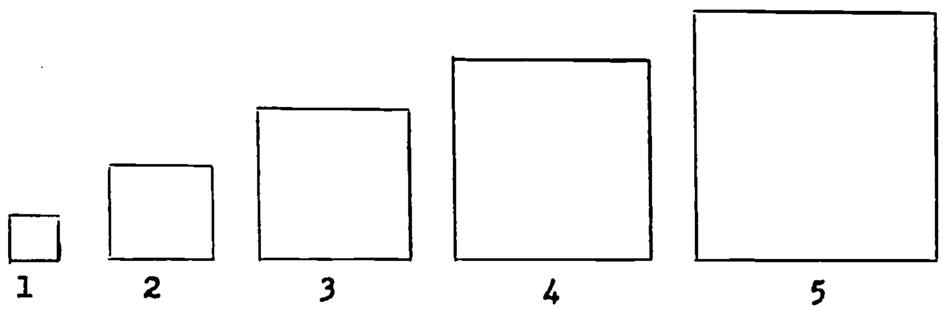
1. 0, 5, 15, <u>20</u>, <u>25</u>, <u>30</u>, <u>35</u>

2. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \underline{\frac{1}{16}}, \underline{\frac{1}{32}}, \underline{\frac{1}{64}}, \underline{\frac{1}{128}}, \underline{\frac{1}{256}}$

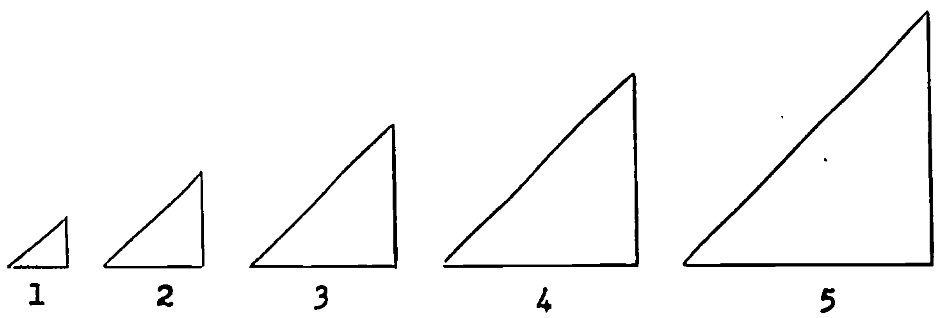
3. 0, 1, 4, 9, 16, <u>25</u>, <u>36</u>, <u>49</u>, <u>64</u> — add successive odd natural numbers (1, 3, 5, 7,...) to 0 or square successive whole numbers

4. 1, 1, 2, 3, 5, <u>8</u>, <u>13</u>, <u>21</u>, <u>34</u> — add previous two terms-Fibonacci numbers

5.



6.



7.

| square numbers | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| perimeter | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 |
| area | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 |

8.

| triangle numbers | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| area | $\frac{1}{2}$ | 2 | $4\frac{1}{2}$ | 8 | $12\frac{1}{2}$ | 18 | $24\frac{1}{2}$ | 32 |

9.

| X | Y | rule |
|---|---|---|
| 1 | 3 | X + 2 |
| 2 | 4 | |
| 3 | 5 | |
| 7 | 9 | |
| 9 | 11 | |

10.

| X | Y | rule |
|---|---|---|
| 1 | 5 | 2x + 3 |
| 2 | 7 | |
| 3 | 9 | |
| 4 | 11 | |
| 5 | 13 | |
| 10 | 23 | |

11.

| X | Y | rule |
|---|---|---|
| 0 | 0 | 4 X |
| 1 | 4 | |
| 2 | 8 | |
| 3 | 12 | |
| 10 | 40 | |

12.

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

Teacher evaluation: students should mention combinations, binomial distribution.

13.

| | | | |
|---|---|---|---|
| 10 | 3 | 15 | 6 |
| 5 | 16 | 4 | 9 |
| 8 | 13 | 1 | 12 |
| 11 | 2 | 14 | 7 |

14. Teacher observation.

15. $-2 \times 2 = -4$

$-2 \times 1 = -2$

$-2 \times 0 = 0$

$-2 \times -1 = 2$

$-2 \times -2 = 4$

$-2 \times -3 = 6$

16. $25 = 1 + 3 + 5 + 7 + 9$

$36 = 1 + 3 + 5 + 7 + 9 + 11$

$64 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$

17. 5th triangular number = 9

7th triangular number = 13

18. 2-last digit is even

3-sum of digits divisible by 3

4-last two digits divisible by 4

5-last digit is zero or 5.

6-last digit is even and the sum of the digits is divisible by 3

9-sum of digits divisible by 9

19.

| 21 | 3144 | 186 | 351 | 450 | 1425 |
|---|---|---|---|---|---|
| $\{3\}$ | $\{2,3,4,6\}$ | $\{2,3,6\}$ | $\{3,9\}$ | $\{2,3,5,6,\}$ | $\{3,5\}$ |

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| | |
|---|---|
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| Flow Chart II | Similarity |
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